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THE PRESENT STATE OF THE QUESTION OF TURBULENT ELECTROCONDUCTIVITY IN PLASMA AND CERTAIN QUESTIONS RELATED TO THE DYNAMICS OF THE MAGNETOSPHERE. I

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ABSTRACT. Phenomena related to plasma turbulence in the circumsterrestrial region of space are considered, and concepts related to turbulent electroconductivity are surveyed. Laminar and turbulent electroconductivity of a plasma are discussed. Relations for a turbulent current are estimated, and theoretical concepts related to a plasma in a strong electric field are presented.

Introduction

The present survey deals with certain phenomena related to plasma turbulence in the circumterrestrial region of space. According to present-day concepts, circumterrestrial space, due to the Earth's magnetic field, is physically speaking a gigantic plasma trap which intercepts the flux of plasma coming from the Sun. An investigation of the structure and dynamics of this geomagnetic plasma trap has been conducted for a relatively long time, and it has picked up momentum with the development of direct means of space research, such as artificial satellites and interplanetary space probes. /1*

The theoretical concepts of the structure and dynamics of the geomagnetic cavity until recently were based on the laminar model of circumterrestrial plasma. These concepts are completely analogous to the concepts of laminar

*Numbers in the margin indicate the pagination in the original foreign text.

stable plasma involved in thermonuclear reactions which held sway over the minds of scientists for over a decade.

When it had become clear during recent years that the numerous forms of instabilities in thermonuclear plasma, discovered theoretically and experimentally, leave hardly any hope for producing laminar high-temperature plasma, there has been a strong outburst of research activity in the area of turbulent plasma. The objective of that research has been to predict, explain, and describe plasma phenomena and properties which appear as a result of instabilities.

The mathematical apparatus used by theoreticians in this field by and large corresponded to the so-called weak nonlinearity approximation in which — when oscillatory motion was discussed — nonlinear terms in the perturbation would remain. Thus, it was possible to use expansions in powers of /2 the amplitude of perturbation fields arising with the development of instabilities. The present state of nonlinear plasma theory makes it possible to describe weakly turbulent quasistationary states of plasma resulting from its instabilities, and to shed light on the radically new effects occurring in those states. Those include, for example, collective interactions, changes of dispersive properties, anomalous diffusion, wave flux braking.

Studies of cosmic plasma proceeded in parallel with those investigations. It is obvious that, in the rarefied plasma occurring in circumterrestrial space, many phenomena can be described using laminar concepts. However, it is clear that an increasingly wide range of phenomena in the physics of circumterrestrial plasma can be interpreted only on the basis of concepts involving a developed turbulence, to say nothing of the fact that the boundary region between interplanetary space and the magnetosphere of the Earth has a pronounced quasistationary turbulence to which laminar concepts are hardly generally applicable.

A separation of turbulent regions, and a determination of the level of turbulence, are also important in solving problems of the acceleration of particles in the magnetosphere and in understanding the dynamics of the Earth's radiation belts, auroras, and other phenomena.

Swift in [1] has proposed considering the region of the magnetosphere close to the zone of auroras as a region of ionic-sound turbulence. The following experimental data:

1) the fact that electrons emitted during auroras gain their energy directly during their emission [2], /3
2) during auroras, currents of intensities up to 10^5 A appear in the ionosphere (it is not proven where they are formed),
3) the presence in the aurora zone of a high level of ultralow-frequency waves indicated by both terrestrial and satellite data, according to [1],

can be explained if we assume that in the Earth's magnetosphere a separation of charges may occur caused by the electric fields perpendicular to the geomagnetic field. In an almost collision-free plasma of the magnetosphere, the electric fields produce currents along the magnetic force lines, uniting space charge regions across the conducting ionosphere. When the currents are of sufficient strength, there arises in the plasma an ionic-sound instability (manifested on the ground as ultra-low frequency noise) which results in the appearance of "collective interactions" of particles with waves, and consequently in an anomalous resistance of the magnetosphere plasma. As a result of this type of turbulence in a plasma which is collision free (in the sense that there are no pairwise collisions) electrodynamic heating of electrons to kilovolt energies may take place. A flux of such "hot" electrons is obviously capable of producing auroras and other accompanying phenomena.

It should be emphasized that the paper referred to, written in 1965, made use of the concepts and estimates related to turbulent flow which were developed by Buneman [3] in 1959 when the theory of turbulent plasma was in its infancy. From the vantage point of present concepts about turbulent plasma, one can say that [1] is accurate only as to its qualitative, but not quantitative (even in its estimates), concepts about the deceleration of electronic streams colliding with ionic-sound pulsations of high amplitude.

The estimates of turbulent electroconductivity of the type $\sigma = 50\omega_{pi} =$ /4 $const$, that were used by Swift, from the present point of view may be applicable only to the very beginning of the development of an instability in an isothermic plasma with a current.

In quansistationary states which may last very long the turbulent electroconductivity $\sigma \sim 1/E$, and the current density is constant within a wide range of variation of the electric field if E is smaller than some E**. For stronger fields $j \sim \sqrt{E}$ (and $\sigma \sim 1/\sqrt{E}$, respectively.

For this reason it is timely and advisable to make a survey of present concepts related to turbulent electroconductivity and an application of these concepts to both the problem posed in [1] and to a number of other problems occurring in the physics of the magnetosphere in which it is necessary to know the function $\sigma(E)$.

§ 1. Laminar and Turbulent Electroconductivity of Plasma

The questions related to the turbulent electroconductivity of plasma have been quite intensively studied during recent years by a number of authors [4 - 11].

The difficulty of the problem consists primarily in the fact that the existence of quasistationary turbulence in a constant electric field is apparently of short duration. It is possible that there are several quasi-stationary turbulent states, each of which is characterized by its own time scale.

Therefore, if one is interested in the anomalous resistance of plasma in a strong electric field (having in mind any kind of formula or even an estimate, replacing σ, determined by interactions), it is necessary to understand clearly that the quantity σ (turb.) depends on the field E in which the plasma is located, and on the time. Therefore, we clearly have to speak directly of a function $j = j(E)$. In addition, since the turbulence level depends to a large extent on the boundary conditions, the function $j = j(E)$ /5 will also substantially depend on them. In this case, the field may E must be understood as the real field at each point. This field may differ significantly from the initial "external" given field at $t = 0$. It must be noted that here the relationship $j = j(E)$ refers to an average taken over a time interval many times longer than the period of oscillations $2\pi/\omega_{pi}$ for the ionic sound wave.

It must be emphasized that we shall consider two cases: involving nonisothermic plasma $T_e \gg T_i$ and isothermic plasma $T_e \approx T_i$. From our point of view, the first case is of greatest importance, since in the second case the turbulization of the plasma in the electric ion invariably results in a lack of isothermicity, and if we are interested in longer turbulence times, we shall deal only with a nonisothermic plasma.

Thus, suppose that an external electric field E is applied to a quasineutral plasma consisting of two kinds of electrons and ions with densities $n_e = n_i = n_0$. For simplicity, we shall consider the plasma to be homogeneous and infinite in extent, and the E field to be homogeneous. If the field is sufficiently small, the plasma is characterized by the usual collision resistance. Just as was done, for example, in [12], to get the feel for things we shall give an elementary analysis for this simple case.

Under the influence of the electric field, the electrons gain a directional velocity with respect to the ions, and should the collisions be absent the electron velocity would grow linearly with time, $v = \frac{eE}{m}t$. Assuming that during each collision an electron completely loses its velocity, we obtain the average speed

$$u = \frac{eE}{m_e}\,\frac{\tau}{2}, \qquad (1.1)$$

where $\tau_e = 1/\nu_{ei}$ is the mean time between two electron collisions. /6

We shall estimate the frequency of collisions. Since we are in fact considering collisions with a substantial change of the momentum, in this type of collisions potential energy e^2/r is on the order of the kinetic energy $mv^2/2$, whence $r \propto \frac{2e^2}{mv^2}$, and the cross section is

$$\sigma = \pi r^2 \simeq \frac{4\pi e^2}{m^2 v^4}. \qquad (1.2)$$

Then the collision frequency is

$$\nu_{ei} \simeq n v \sigma = \frac{4\pi n e^4}{m^2 v^3}. \qquad (1.3)$$

When the velocity of electrons is much smaller than the mean thermal velocity $u \ll v_{Te}$, we can substitute v in Equation (1.3) instead of v_{Te} to obtain

$$\nu_{ei} = \frac{4\pi n e^4}{m^2 v_{Te}^3} = \frac{\omega_{pe}^4}{4\pi n_0 v_{Te}^3} = \frac{\omega_{pe}}{4\pi (n\lambda_e^3)}$$
$$\left(\lambda_e = \frac{v_{Te}}{\omega_{pe}}\right).$$

This quantity does not depend on the field E and the velocity u. More accurate calculations yield

$$\nu_{ei}^0 = \frac{\omega_{pe} \ln \Lambda}{4\pi n \lambda_e^3}. \qquad (1.4)$$

where $\omega_{pe} = \sqrt{4\pi e^2 n / m_e}$, n -is the number of electrons per unit volume, /7
$\lambda_e = v_{Te} / \omega_{pe}$ is the Debye electron length, $\ln \Lambda$ is the so-called Coulomb logarithm ($\ln \Lambda \sim 10$).

Substituting ν^0_{ei} according to Equation (1.4) in Equation (1.1), we obtain

$$u = \frac{1}{2} \frac{eE}{m_e} \frac{4\pi n \lambda_e^3}{\omega_{pe} \ln \Lambda}$$

and consequently, Ohm's law in the laminar case is

$$j = neu = \frac{ne^2 E}{m_e} \frac{1}{2} \frac{4\pi n \lambda_e^3}{\omega_{pe} \ln \Lambda} = \sigma_{\mathrm{lam}} E, \qquad (1.5)$$

where

$$\sigma_{\mathrm{lam}} = \frac{\omega_{pe}^2}{8\pi \nu^0_{ei}} = \frac{\omega_{pe}^2}{8\pi} \frac{4\pi n \lambda_e^3}{\omega_{pe} \ln \Lambda}.$$

It may be stated that in the laminar case the force acting on an electron, eE, is balanced by the force of "friction" against ions equal to the loss of momentum per unit time

$$f_{\mathrm{Tp}} \simeq \frac{\Delta p}{\Delta t} \simeq \frac{2u_0 m_e}{\tau_{ei}} \simeq 2u_0 m_e \nu_{ei},$$

i.e.,

$$eE = 2m_e u_0 \nu_{ei}. \qquad (1.6)$$

Now we shall turn to the question: up to what values of E is the linear /8
Ohm's law (1.5) valid? It is easy to see from Equation (1.6) that the linear

Ohm's law is valid as long as $\nu_{ei} = const$, i.e., under the condition $u \ll v_{Te}$. When $u \gtrsim v_{Te}$, we must replace v in Equation (1.3) with u, and then ν_{ei} is no longer constant, and depends on the directional velocity as

$$\nu_{ei} \sim \frac{1}{u^3}.$$

The above relation means that the force of "friction" of an electron against ions is $2m_e u \nu_{ei} \sim 1/u^3$, and it falls off with increasing velocity u.

Thus, the force of "friction" of an electron against the ions for $u \ll v_{Te}$ is directly proportional to u. For $u \sim v_{Te}$, the force of "friction" passes through a maximum; for $u > v_{Te}$ it falls off.

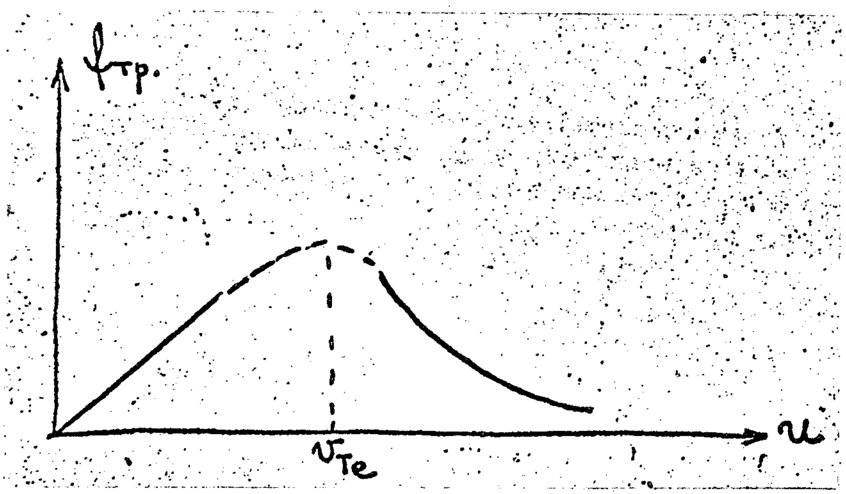

Figure 1 a.

The electric field E_D for which $f_{Tp.} = f_{Tp}^{max}$, is easy to estimate by /9
substituting $u \simeq v_{Te}$ in (1.6), and according to Equation (1.3), $E_D = 2e/\lambda_e^2$. More accurate calculations result in a slightly larger coefficient

$$E_D = \frac{e}{\lambda_e^2}\frac{\ln\Lambda}{2} = \frac{T_e \ln\Lambda}{8\pi e n \lambda_e^4}. \tag{1.7}$$

If the electric field $E > E_D$, dynamic equilibrium of the electric field E and the forces of friction due to pair-wise collisions is impossible. If one only considers pair-wise collisions, then for $E \gtrsim E_D$ all the electrons in the plasma in an electric field should be freely accelerated, and the current should grow linearly with time. However, in practice this does not occur since an ionic-sound instability occurs in a plasma for $T_e \gg T_i$. The increasing ionic-sound fluctuations interacting with the particles decelerate them, i.e., the dominant role begins to be played by collective interactions instead of pairwise electron-ion collisions.

In practice, in a nonisothermic plasma the instability of ionic-sound waves begins for $u \sim C_s$ (where C_s is the speed of the ionic sound), i.e., for fields much smaller than E_D.

§ 2. Estimated Relationships for a Turbulent Current /10

The instability involving ionic-sound waves arises when the electric field $E > E^*$ is such that the velocity of electrons amounts to $u_{min} \approx C_s$ (a more accurate expression can be found, for example, in [5]). Then

$$\frac{E^*}{E_D} \approx \frac{u_{min}}{v_{Te}} \approx \frac{c_s}{v_{Te}} \approx \sqrt{\frac{m_e}{m_i}}.$$

Later as a result of the development of instability, a quasi-stationary ionic-sound turbulence is formed. Thus for sufficiently long times $t \gg 1/\omega_{pi}$ for fields

$$E > E^* = E_D\sqrt{\frac{m_e}{m_i}} = \frac{e}{\lambda_e^2}\frac{\ln\Lambda}{2}\sqrt{\frac{m_e}{m_i}} \tag{2.1}$$

the plasma is turbulent, and as shown by theory, within a certain interval $E^* < E < E^{**}$, the current density does not depend on E, i.e., we have a plane current-voltage characteristic, and $\sigma \sim 1/E$. As an example let us consider a plasma of the Earth's magnetosphere in the aurora zone:

$$n_e = 10^4 \text{cm}^{-3},\ T_e \sim 1\,\text{keV} = 1.6 \cdot 10^{-9}\ \text{ergs},\ \lambda^2_d = 5{,}5 . 10^4 \text{cm}^2,$$
$$E_D = 1{,}2.10^{-11}\ \text{V/cm},\ E^* = 3.10^{-13}\ \text{V/cm}$$

As we know magnetospheric electric fields along the magnetic lines of force are determined by the potential difference ~ 10 Kv at a distance of $\sim 10\,000$ km, i.e., they are on the order of $\sim 10^{-8}$ V/cm. Therefore, in particular, the magnetosphere currents along the lines of force in the aurora region may certainly have a turbulent character. /11

Thus, suppose that there is a field with $E > E^*$ in a plasma, and the current velocity is $u_0 \gtrsim c_s$. Then an ionic-sound instability begins to develop in the plasma. The accumulated ionic-sound fluctuations of large amplitude interact almost elastically with a majority of plasma electrons, by and large participating only in an exchange of momentum. The energy exchange is insignificant.

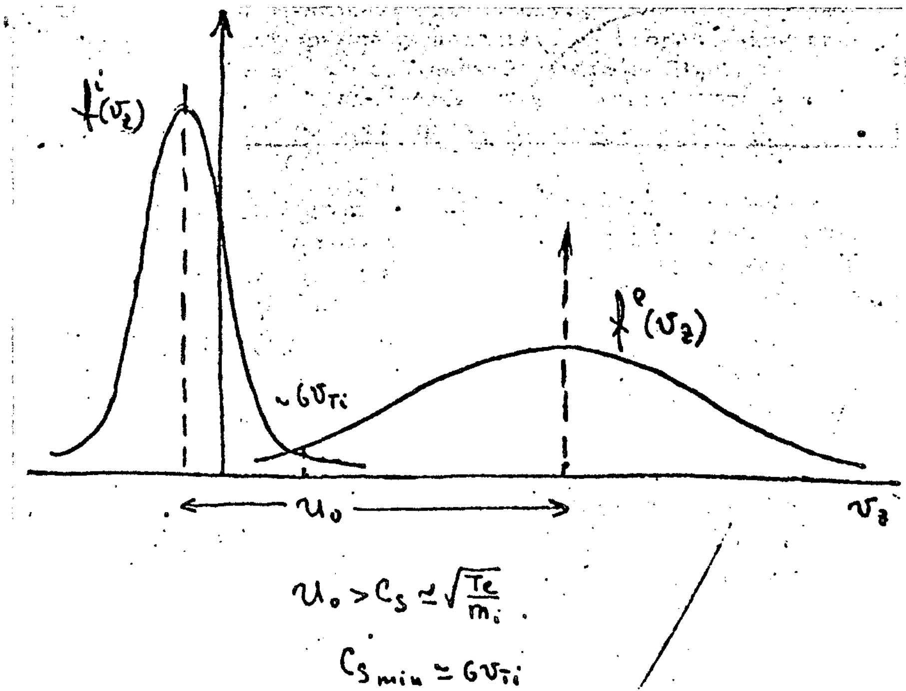

Figure 1a. Distribution functions of electrons and ions in the current plasma.

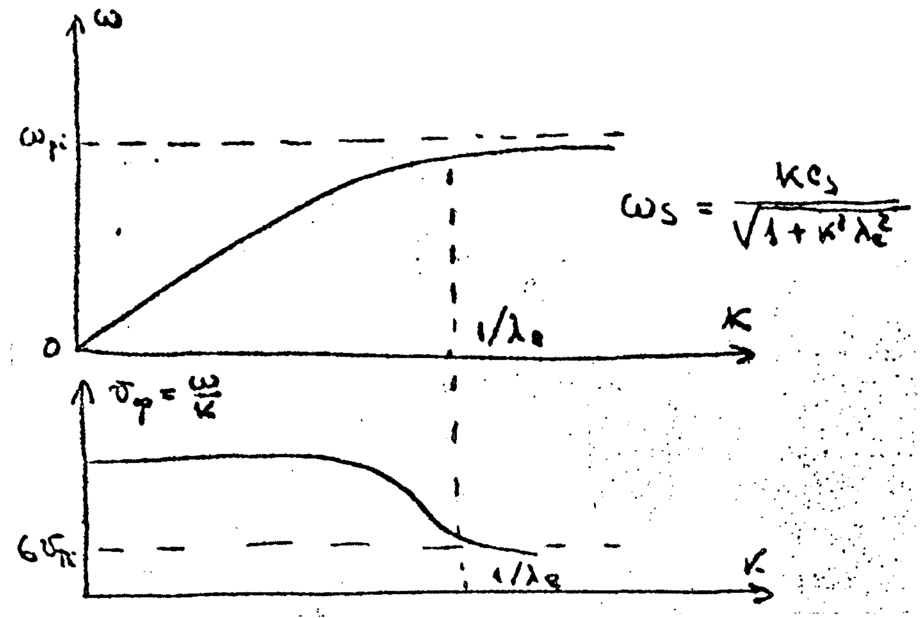

Figure 1b. The variance curve for ionic-sound waves. /12

A wave characterized by ω and $\vec{K}$ is triggered by the so-called resonance electrons whose velocity in the direction $\vec{K}$ is equal to $v_{p} = \omega/K$, i.e., we have a resonance condition $\omega = \vec{K}\vec{v}$. The activation or the attenuation of waves may be understood to be a result of a large number of incidents involving Cherenkov emission and an absorption of ion-sound plasmons by resonance electrons. Let us clarify why the interaction of electrons with ionic-sound plasmons occurs almost elastically — to be more precise, why it occurs with conservation of the energy of electrons in the zero approximation with respect to the parameter ω/Kv_e.

Since in the process of the interaction of electrons with ionic-sound waves $\omega = \vec{K}\vec{v}$, and the phase wave velocity is equal to $c_s = v_{Te}\sqrt{m_e/m_i}$, the resonance electrons moving with a velocity on the order of thermal velocity emit or absorb waves with $\vec{K}$ which is orthogonal to $\vec{v}$ within an accuracy of $\sqrt{m_e/m_i}$ as compared to c. The velocity increase $\Delta\vec{v}$ is obviously parallel to the wave vector, i.e., perpendicular to $\vec{v}$, which means that in a first approximation the energy increase is 0. /1

Since the electrons that gain momentum in the electric field lose it due to scattering from the waves, in the last analysis the wave amplitude increases to a value such that the force of friction of electrons against waves balances the action of the electric field.

If it turns out that in this state the current velocity noticeably exceeds the critical value, then the noise and the force of friction will increase as before, and the velocity u will fall off until it reaches a value near u_0.

As a result, within a fairly wide range of electric field values the current density remains almost constant, and $\sigma \sim 1/E$.

Following [5] we shall write the conditions for the equilibrium of forces acting on the electrons and ions.

From these conditions one can obtain approximate information about the order of magnitude of the energy of ionic-sound plasmons in a quasi-stationary state (assuming it exists):

$$-en\vec{E} = \int \vec{\kappa}\, \gamma_e(\vec{\kappa}) N_{\vec{\kappa}}\, d\vec{\kappa}, \tag{2.1}$$

$$en\vec{E} = \int \vec{\kappa}\, \gamma_i(\vec{\kappa}) N_{\vec{\kappa}}\, d\vec{\kappa}. \tag{2.2}$$

Here we took into consideration the fact that, since the total momentum of the ionic-sound wave is $\vec{P} = \int \vec{\kappa} N_{\vec{\kappa}}\, d\vec{\kappa}$, then for electrons

$$\left(\frac{\partial \vec{P}}{\partial t}\right)_e = \int \vec{\kappa} \left(\frac{\partial N_{\vec{\kappa}}}{\partial t}\right)_e d\vec{\kappa} = \int \gamma_e(\vec{\kappa})\, \vec{\kappa} N_{\vec{\kappa}}\, d\vec{\kappa}.$$

An analogous relation is also valid for ions. It is necessary to emphasize that even though (2.1) and (2.2) imply

$$\int(\gamma_i + \gamma_e)N_{\vec{k}}\vec{k}\,d\vec{k} = 0 \text{ but } \gamma_i = -\gamma_e$$

it by no means follows in a general case.

Let us now consider the density equilibrium of ionic-sound plasmons. The kinetic equation for plasmons is obviously of the form (in a homogeneous plasma)

$$\frac{\partial N^S_{\vec{k}_s}}{\partial t} + \frac{\partial\omega}{\partial\vec{k}}\vec{\nabla}N^S_{\vec{k}} = \gamma_e(\vec{k})N^S_{\vec{k}} + \gamma_i(\vec{k})N^S_{\vec{k}}. \qquad (2.3)$$

In this equation the term $\frac{\partial\omega}{\partial\vec{k}}\vec{\nabla}N^S_{\vec{k}}$ corresponds to the case when the inhomogeneity of plasmon gas in space is important, and it describes a change of $N_{\vec{k}}$ due to the movement of quasi-particles toward the boundary of the turbulent region with a group velocity $C_s = \partial\omega/\partial\vec{k}$.

An estimate of this term yields

$$\frac{\partial\omega}{\partial\vec{k}}\vec{\nabla}N^S_{\vec{k}} \simeq \frac{C_s}{a}N^S_{\vec{k}_s} \simeq \omega_{0i}\frac{\lambda_e}{a}N^S_{K_s}, \qquad (2.4)$$

where a is the size of the inhomogeneity.

For the present, we shall neglect the term (2.4), considering the characteristic lateral dimension of the plasma to be sufficiently large. The first /15 term on the right in (2.3), $\gamma_e N^S_{\vec{k}}$, corresponds to a linear generation of waves by plasma electrons, and is determined by the form of the distribution function f^e. In this case for a Maxwell distribution function

$$f^e = \left(\frac{1}{2\pi v_e^2}\right)^{3/2} n_0 \exp\left\{-\frac{(\vec{v}-\vec{u}_0)^2}{2v_e^2}\right\} \text{ maximum value of } \gamma_e$$

$$\gamma_e^{max} = \gamma_e^0 = \frac{u_0}{v_{Te}}\omega_{0i} \text{ where } u_0 \sim C_s = \sqrt{T_e/m_i}$$

When the time development of the process is considered, in the initial phase of development of an instability $\gamma_e = \gamma_e^{max} = \omega_{oi}(u_o/v_{Te})$, the increment, proportional to $(\partial f/\partial v)$, decreases almost to zero due to a distortion of the electron distribution function resulting from an interaction with waves.

Finally, the last term in (2.3) corresponds to two processes: $\gamma^i = \gamma^i_{nonlin.} + \gamma^i_{res}$. γ^i_{res} corresponds to a linear absorption of waves by resonance ions (having a velocity $v_i \gg c_s$) and a nonlinear scattering of ion-sound waves off the ions, which can be estimated as [12, 5]

$$\gamma^i_{nl}{}^* \sim \omega_{oi} \frac{W}{nm_ic_s^2} = \omega_{oi} \frac{\int \omega_s N_k dk}{nT_e} \qquad (2.5)$$

As far as a linear absorption of waves by resonance ions is concerned, then within only a framework of quasilinear effects accordint ot [5] we have (n' is the density of resonance ions, T' is the temperature)

$$\begin{gathered} \gamma^i_{res} \sim f\left(\frac{\omega}{k}\right) \sim n'/T_i'^{3/2}; \quad (n' = const) \\ \frac{d}{dt}(n'T') = \gamma^i_{res} = \alpha T_i'^{-\frac{3}{2}}, \quad \alpha \int dt = n' \int dT_i' T_i'^{3/2} \\ n \sim n' T_i'^{5/2}, \quad T_i' \sim t^{2/5} \\ \gamma^i_{res} \sim t^{-2/5}. \end{gathered} \qquad (2.6)$$ /16

Thus, γ^i_{res} decreases with time as $t^{-2/5}$. Therefore, it is necessary in the first place to consider a process with a nondecreasing increment, that is, nonlinear scattering by the ions. It must be noted that the scattering results primarily in a change of direction of the momentum of a quasi-particle and in an isotropization.

Furthermore, when considering nonlinear scattering by ions, Equation (2.6) in turn loses its validity, since one must take into consideration an increase in the number n' of resonance ions and their heating, along with heating of

* Translator's Note: nl designates nonlinear.

the basic nonresonance mass of the ions.

These latter processes, incidentally, result in the appearance of escaping ions.

If one assumes that the basic contribution to the quantity γ^i is made by nonlinear scattering by ions, and sets $\gamma^i = \gamma^i_{nonlin}$ for a boundless plasma in the quasi-stationary case, one then obtains

$$-\gamma_e = \gamma^i = \gamma^i_{nl} \tag{2.7}$$

From Condition (2.2) we have, using $\gamma^i = \gamma^i_{nl}$ /17

$$enE \simeq \gamma^i_{nl} \frac{W}{c_s} = \omega_{pi} \frac{W}{nT_e} \frac{W}{c_s}. \tag{2.8}$$

Hence we find that in the quasi-stationary turbulent mode in regions II and III (Figure 2) the total energy density of turbulent ionic-sound pulsations is

$$W = nT_e \left(E/\sqrt{8\pi n T_e} \right)^{1/2}. \tag{2.9}$$

In this case $\gamma_e = \gamma_{nl} = \omega_{pi} W/nT_e = \omega_{pi}\left(E/\sqrt{8\pi nT}\right)^{1/2}$, i.e., γ_e and γ_{nonlin} increase with a growing field E.

It is necessary to point out here that simple physical considerations (which are confirmed by detailed calculations) make it necessary to consider separately two intervals of the electric field values.

If one considers a change in the picture of the turbulent quasi-stationary state depending on E, one can see that from $E = E^*$ to $E = E^{**}$ the turbulent energy varies as $W \sim \sqrt{E}$.

In this connection, we have the increment $\gamma^i = \gamma^i_{nonlin}$.

Here γ_e, which remains equal to γ^i with an increase in E, increases due to a decrease in the quasi-linear effect, i.e., a decrease of the reversing action of ionic-sound waves on the electron distribution function.

The increase of γ_e continues up to $\gamma_e^0 \approx \omega_{pi} c_s^0 / v_e \simeq \omega_{0i} \frac{u_e}{v_{Te}}$, corresponding to $E \approx E^{**}$. Here in the interval $E^* < E < E^{**}$ the velocity u is practically constant and equal to $u \approx c_s$. Consequently, $j \approx enc_s = const$. /18

The corresponding section of the plane current-voltage characteristic (II) for the plasma is shown in Figure 2. Section I in the diagram corresponds to the usual conductivity in the nonturbulent plasma.

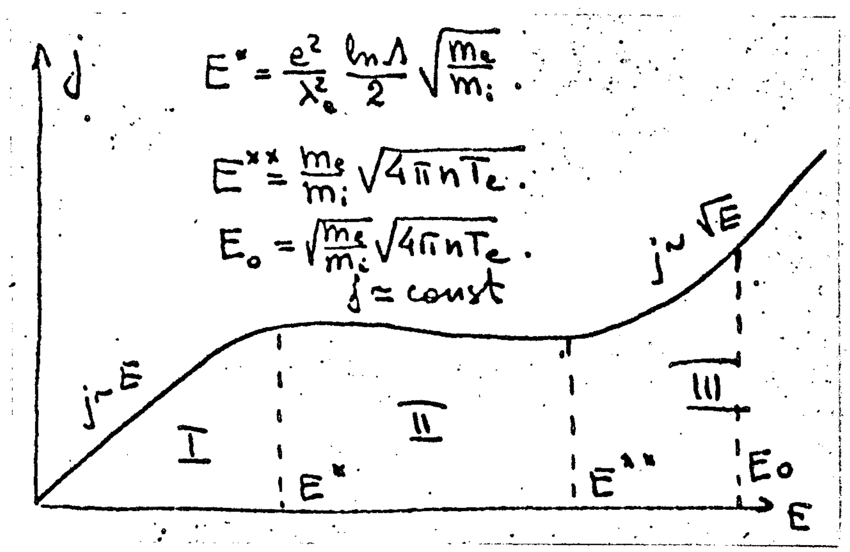

Figure 2

We note, incidentally, that the usual semi-qualitative considerations are not applicable in Section II. These arguments yield an estimate for the turbulent electroconducitivity, when one writes:

$$j = \sigma E, \qquad \sigma = \omega_{pe}^2 / 8\pi \nu_{st},$$
$$\nu_{st} = \nu^{\text{turb.}} \sim \frac{w}{nT} \omega_{pi}$$ /19

and, consequently, $j \sim \sqrt{E}$. Considerations of this type are applicable only for large values of $E > E^{**}$.

The inapplicability of the above considerations in Section II is related to: (a) the strong anisotropy of the ionic-sound turbulence, and (b) the strong reversing effect of turbulence on the portions of the electron distribution function that are responsible for the buildup of waves.

It should be emphasized that the deceleration of the electron stream is most effectively achieved by waves that propagate normally to the stream, and the energy density of such waves in Section II is considerably smaller than the energy density of the waves propagating in a direction parallel to the electron stream.

An increase in E is accompanied by an increase in both the turbulent energy $w \sim \sqrt{E}$ and the isotropic turbulence.

The velocity u is maintained near a value $u \approx c_s$ with increasing E, because the equilibrium of forces (2.1) acting on the electrons is maintained due to an increase of $\gamma_e \sim \sqrt{E}$, caused by a decrease in the quasi-linear effect, and due to an increase of the turbulence isotropy (i.e., due to an increase in the relative number of waves propagating normally to the current).

The plane current-voltage characteristic continues up to values $E \approx E^{*}$. E^{**} is determined by the field at which the energy density of the noise w, needed to keep u close to $u_0 \simeq c_s$, becomes so large that

$$\gamma_{nl\,i} \gtrsim \gamma^{0}_{(lin)e} \simeq \omega_{pi} \frac{u_0}{v_{Te}} \qquad (2.8)$$

Then to maintain noise at the necessary level there must be an increase in the linear increment due to an increase of the current velocity. For $E > E^{**}$ the current-voltage characteristic ceases to be planar (Section II in Figure 2).

Let us consider the condition $\gamma^{**}_{\text{nonlin}} \simeq \gamma^{0}_{\text{lin}\,e}$; substituting /20

$$\gamma^{**}_{nl} = \omega_{pi}\frac{W^{**}}{nT}, \quad W^{**} = nT_e\left(\frac{E^{**}}{\sqrt{4\pi n T_e}}\right)^{1/2},$$

we shall obtain

$$E^{**} = \frac{m_e}{m_i}\sqrt{4\pi n T_e}. \tag{2.9}$$

As far as the current density in the region $E > E^{**}$ is concerned, it can be found from the following considerations. Considering that $\gamma_e \simeq \omega_{pi}\frac{u}{v_{Te}}$, setting $\gamma_e = \gamma^{i}_{\text{nonlin}}$, we obtain

$$u = v_{Te}\frac{W}{nT_e}; \tag{2.10}$$

$$j = enu = enc_s\left(\frac{E}{\sqrt{4\pi n_0 T_e}}\,\frac{m_i}{m_e}\right)^{1/2}. \tag{2.11}$$

Writing Ohm's law in the usual form for a longitudinal current, we obtain

$$\vec{J} = \sigma\vec{E}$$

$$\sigma = \left(\frac{m_i}{m_e}\right)^{\frac{1}{2}}\frac{enc_s}{(4\pi n_0 T_e)^{1/4}\sqrt{E}}.$$

Thus, for $E > E^{**}$ we have $\sigma \sim 1/\sqrt{E}$. We shall give some estimates of /21 E^{**} as applied to the Earth's magnetosphere. Since in this case $n_0 = 10^{1+5}$ cm^{-3}, $T_e = (1+10^4)$ eV, we find that $E^{**} = 2\times10^{-(2+8)}$ V/cm.

Near the Earth, where $n_e \approx 10^4 \mathrm{cm}^{-3}$ for $T_e \approx 1\,\mathrm{keV}$, $E^*/\approx 2.10^{-3}$ V/cm. Measurements of the electric field in Earth's magnetosphere give, for instance, $E \approx 3.10^{-4}$ V/cm . [48]

Thus we can see that in fact both Section II and III may be realized in the magnetosphere.

The quasi-stationary turbulent states, described here, are in addition characterized by a constant growth of the temperatures of the electron and ion gases. These processes are determined, in addition to the equilibirum conditions, by the following equations [5]:

$$enEu = \frac{d}{dt}\frac{3}{2}(p_e + p_i), \tag{2.12}$$

$$\frac{d}{dt}\frac{3}{2}p_i \approx \int \gamma_i N_k \omega \, d\vec{k} \tag{2.13}$$

(p_i and p_e are the pressures of the ion and electron gases, respectively), i.e., roughly speaking, the heating is proportional to the time.

A detailed analysis, done in [5, 8] shows that after a sufficiently long time the ratio T_e/T_i tends to a constant on the order of 10.

The questions related to the escaping electrons and ions remain to a large extent unexplained. Within the framework of the quasi-linear theory, the phenomenon of the escape of electrons [4] and ions [8] can be important in the dynamics of a further development of the quasi-stationary states, and for the /22
resulting current-voltage characteristic. However, in reality, there obviously exist other mechanisms responsible for a deceleration of fast particles that go beyond the framework of the interactions with the ionic-sound waves discussed here.

Therefore, if the other possible mechanisms are considered, then of course it must be expected that the phenomenon of escape will be of little significance.

Now several words will be said about a possible situation when the term $\frac{\partial \omega}{\partial \vec{k}} \vec{\nabla} N_k^s$ is important. A situation of this type can in principle be achieved in the magnetospheric turbulent currents which were mentioned in the introduction.

It is well known that the arcs of the aurorae often have the form of fairly thin surfaces of thickness on the order of 1 km near Earth, and in length amounting to hundreds and thousands of kilometers [32]. Therefore, in certain cases the term $\frac{\partial \omega}{\partial \vec{k}} \vec{\nabla} N_k^s$ can be important in explaining certain geophysical phenomena.

Thus, let us assume that a linear generation of ionic-sound plasmons in the plasma is mainly compensated for by their escape across the boundary of the turbulent plasma:

$$\frac{\partial \omega}{\partial \vec{k}} \nabla N_k^s \simeq N_k \omega_{pi} \frac{\lambda_e}{a} = \gamma_e N_k. \tag{2.14}$$

and then $\gamma_i = \gamma_{nl}^i \ll \gamma_e = \gamma_{es} \equiv \omega_{pi} \frac{\lambda_e}{a}$.

(γ_{nonlin} is insignificant in the balance of N_k).

In this case, however, the level of turbulent energy is determined by the equation of the equilibrium of forces acting on the ions (i.e., in this process γ_i^{nonlin} is important), i.e., $W = nT_e (E/\sqrt{4\pi n T_e})^{1/2}$. For $E \ll E^{**}$ the current-voltage characteristic is obviously planar as before /23
(since $\gamma_i \ll \gamma_e < \gamma_e^0$).

Although the case of very large $c_s \nabla N_k$ has not yet been studied in detail, it is reasonable to assume that a stable state for which simultaneously the Conditions (2.14) and the conditions of the equilibrium of forces acting on electrons and ions (2.1), (2.2) can be stably satisfied, is scarcely possible.

It can be expected that generally speaking there will not be turbulent stationary states of current plasma when the relative role of the term $\frac{\partial\omega}{\partial \vec{k}} \vec{\nabla} N_k^s$ is enhanced, i.e., the waves will not escape the turbulent region. To this it must be added that obviously at the boundary of a "tube" of a turbulent current where the term $\frac{\partial\omega}{\partial \vec{k}} \vec{\nabla} N_k^s$ is always important, the average current velocities of electrons and ions may be much higher when the processes at the boundary are nonstationary.

Development of a turbulent nonstationary process at the lateral boundary of a current tube in space, as we assume, takes place during decay of the aurora when — because of the groove instability — the luminous surface of the aurora becomes thinner, forming wrinkles, "draperies", increasing the relative surface of current plasma, and, consequently, the volume of the plasma, where the term describing the escape of plasmons $\frac{\partial\omega}{\partial \vec{k}} \nabla N_k$ is important. In this connection a sharp burst of ultralow-frequency waves and the current takes place during decay of aurora.

We must emphasize that the criterion of a turbulent quasi-stationary state may be written in the form $\gamma_{\text{nonlin}} \gg \gamma_{\text{es}}$. Thus when

$$\frac{W}{nT_e} \lesssim \frac{\lambda_e}{a} \tag{2.15}$$

there a breakdown of the turbulent quasi-stationary state occurs because of the escape of plasmons. The last condition can also be written as

$$E/\sqrt{8\pi n T_e} \leq \lambda_e^2 a^{-2} \tag{2.16}$$

If, as an example, we take in the region of aurora $a \sim 2.10^4$ cm, $a\lambda_e \sim 2.10^2$ then $\lambda_e/a \sim 10^{-2}$. In this case decay of the turbulent current, accompanied by a short nonstationary phase of a free acceleration of particles in the electric field, will occur for $E \sim 0{,}2\, E^{**}$, i.e., in the plane region of the current-voltage characteristic.

3. Modern Theoretical Concepts related to a Plasma in a Strong Electric Field /24

We proceed now to survey modern theoretical notions about behavior of a plasma in a strong electric field.

First of all it will be noted that the case $E < E^*$ — when the constant velocity of the electron drift relative to ions is a result of braking — was studied a long time ago, and the corresponding theory has been presented in numerous monographs and textbooks (for example, [13, 14, 15]).

Under the condition $E > E^*$ an anomalous large resistance of a non-isothermic plasma was observed many times experimentally in the electric field, i.e., the electrons were not freely accelerated [18, 20]. To explain this phenomenon a number of authors have developed a nonlinear theory which implies that the anamalous resistance of a plasma is a consequence of the scattering of electrons by ionic-sound noise of great amplitude, i.e., it occurs during collective interactions [23].

As already indicated in the preceding section, in relation to the ionic-sound noise a nonisothermic plasma ($T_e \gg T_i$) is unstable if the velocity is greater than the velocity of the ionic sound. According to [4, 11] the problem under consideration can be described by a system of equations for the electron and ionic distribution function f^e и f^i and for the mean square of a component of the Fourier potential of the ionic-sound noise $|\varphi_k|^2$ (or the density of the number of the ionic-sound plasmons N_k).

The starting system of equations, according to [4] has the form

$$\frac{\partial f^e}{\partial t}+\frac{e}{m}E\frac{\partial f^e}{\partial v_z}=\frac{\partial}{\partial v_\alpha}D_{\alpha\beta}\frac{\partial f^e}{\partial v_\beta}, \tag{3.1}$$

$$D_{\alpha\beta}=\frac{\pi e^2}{m^2}\int \kappa_\alpha\kappa_\beta|\varphi_\kappa|^2\delta(\omega-\vec{\kappa}\vec{v})\,d\vec{\kappa}, \tag{3.2}$$

$$\begin{gathered}\frac{\partial N_{\vec{\kappa}}}{\partial t}=\gamma_{\text{lin}}N_{\vec{\kappa}},\\ N_{\kappa,\theta}=\kappa^2\left(\frac{\partial\varepsilon}{\partial\omega}\right)|\varphi_\kappa|^2/8\pi,\end{gathered} \tag{3.3}$$

$$\begin{gathered}\gamma_{\text{lin}}=\frac{\pi}{2}\frac{\omega^3}{\kappa^2}\frac{M}{mn}\int\vec{\kappa}\frac{\partial f^e}{\partial\vec{v}}\delta(\omega-\vec{\kappa}\vec{v})\,d\vec{v}-\\ -\frac{\sqrt{\pi}\,\omega^4}{\kappa^3 c_i^3}\exp\left[-\frac{\omega^2}{\kappa^2c_i^2}\right],\end{gathered} \tag{3.4}$$

where

$$\omega=\frac{\kappa c_s^0}{\sqrt{1+\kappa^2\lambda_e^2}};\quad \lambda_{De}^2\equiv\lambda_e^2=\frac{v_e^2}{\omega_{pe}^2}=\frac{T_e}{m_e\omega_{pe}^2},\quad c_s^0=\sqrt{\frac{T_e}{M}}$$

The starting system of equations does not take into account pairwise collisions. The velocity distribution of electrons is assumed to be close to the Maxwell distribution. The external magnetic field is zero; the magnetic field of the current is negligibly small.

Here $f(\vec{v},t)$ is the electron distribution function, $D_{\alpha\beta}$ is the diffusion coefficient of electrons by ionic-sound waves, γ_κ is the increment of the increase of a κ^{th} harmonic of ionic-sound waves. References [4.9] when considering the time-dependent problem do not take into account the time dependence of the ion distribution function. Thus the system, (3.1) - (3.4) describes the process of electron deceleration by ionic-sound waves for $t < t_0 = \sqrt{T_e M}/eE$ (i.e., up to a time when the ions reach a velocity equal to the velocity of sound $v_i^0 = at_0 = (eE/M)t_0 = C_s^0$).

As we can see from the equations, a buildup of waves with parameters ω and $\vec{k}$ is achieved with the help of resonance particles whose velocity in the direction of the wave vector is equal to the phase velocity of the wave $\omega = \vec{k}\vec{v}$. The variation of the electron distribution function is related both to the presence of the electric field, and to a deceleration by the ionic-sound noise.

If the electric field E_0 is applied initially, the electrons begin to accelerate, and as soon as their velocity becomes greater than C_s, ionic sound instability begins to develop: initially exponentially, and then the increment must become zero since the work done by the electric field on the particles of the plasma does not increase faster than t^2, and in the final analysis the energy of oscillations is obtained from the work done by the field.

The first initial stage of instability development, which is most difficult to solve analytically, was investigated in detail by Field and Fried [9] who used a computer. Their paper shows that the problem is essentially not one-dimensional, and although the propagation of waves, parallel to the applied field, plays an important role initially, as time goes on the waves begin to propagate primarily at finite angles with the z axis along /27
which the electric field is directed. In other words, the diffusion described by Equation (3.1) occurs basically at an angle in the velocity space.

We shall give the computer results from [9] for the time-dependence of the mean velocity:

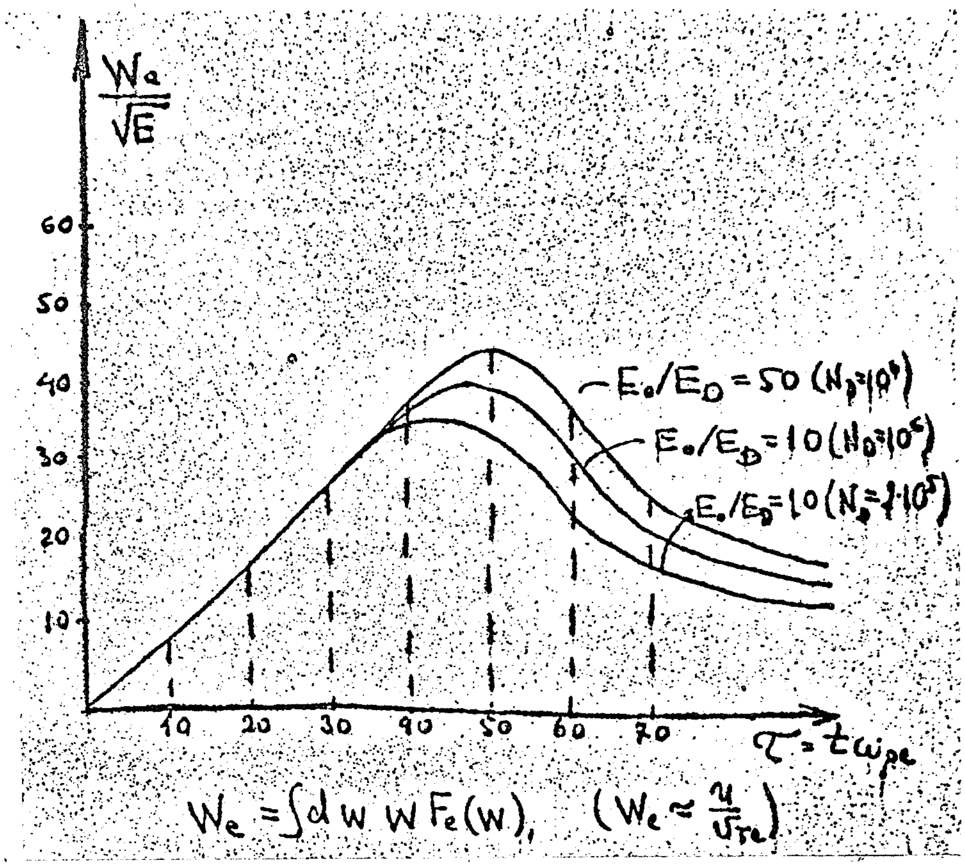

Figure 3

From this we see that the mean velocity and the current attain a maximum, and the heating of electrons is seen to continue afterwards.

Qualitatively this type of velocity behavior can be explained by the fact that the diffusion coefficient D in the velocity space increases sharply up to that time in the velocity region $v \simeq 0$.

The second stage of the process was studied in detail in [4] on the basis of an assumption about the existence of an "almost quasi-stationary state" when the force of dynamic friction does not permit a large portion of the electrons to accelerate freely. In the same paper a solution was found for the starting system of equations, corresponding to a state in which the waves are stable for all angles $\Theta' > \pi/2$ $(\gamma_{\kappa,\theta'} < 0)$, and the increment $\gamma_{\kappa,\theta'} = 0$ for $\pi/2 \geq \theta' \geq 0$. The wave distribution function $N_{\kappa,\theta'} = (\frac{\partial \varepsilon}{\partial \omega}) \kappa^2 |\varphi_\kappa|^2 / 8\pi$ is approximated by a formula $N_{\kappa,\theta} = N_\theta \delta(\kappa - \kappa_0)$, where κ_0 is determined by the conditions $\gamma(\kappa_0, \Theta) = 0$ and $\frac{\partial}{\partial \kappa} \gamma(\kappa_0, \Theta) = 0$ for the angles Θ which are not very close to zero. However, for angles Θ close to zero there are no stationary solutions, and in this connection the important result was obtained that the energy of the ionic-sound waves increases proportionately to the time $W = ct$. A detailed analysis of the role of escaping electrons has shown that their contribution to the increment is insignificant. The solution obtained corresponds to an "almost constant" saturation current $j \sim e n_0 c_s$. The work of the electric field during a time

$$\sqrt{\frac{m_e}{m_i}} \frac{m_e c_e}{e E_0} = t_{min} < t < t_0 = \frac{m_e c_e}{e E_0} \sqrt{\frac{m_i}{m_e}}$$

(i.e., until the ions achieve a velocity $v_i \sim c_s = \sqrt{T_e/m_i}$) is expended /29 on an increase in the energy of oscillations and the heating of electrons.

Furthermore (within the framework of the theory) due to their heating the electrons should start escaping, and the current should increase with the time. The density of the oscillation energy increases proportionally to the time, and for $t = t_0$ it reaches nT_e.

We should emphasize once again that within the framework of [4] the ionic-sound instability limits growth only in the time $t < t_0$, and a constant segment of the current-voltage characteristics occurs also when $t < t_0$. Furthermore, the solutions under consideration — which are not, strictly speaking, quasi-stationary, because of escaping electrons — are valid only if the time that it takes for an instability to develop, $t_i \sim 1/\gamma_0 \sim (\omega_{pi} c_s / v_e)^{-1}$ is less than t_0, i.e.,

$$\frac{E}{\sqrt{4\pi n T_e}} \ll \sqrt{\frac{m_e}{m_i}}.$$

Furthermore, among the most important papers that develop the theory of plasma turbulence in a strong electric field we must include a paper written by Kovrizhnykh [7].

Reference [7] takes into account both the interaction of electrons with the ionic-sound noise and the pairwise collisions ([6] is devoted to an investigation of an analogous problem in a one-dimensional model), and as a result obtains a corresponding analytic solution for a quasi-stationary spectrum of ionic-sound noise. Reference [7] obtains explicit equations for the time-dependence of the mean kinetic energies of plasma electrons and ions. The equations imply that the presence of ionic absorption results in intense heating of the ion component of the plasma whose rate is proportional to the value of the external field E. Since the collisions were taken into account in [7], it was possible to find the value of the electric field below which the number of the escaping electrons is negligibly small. /30

Thus, in [7] the starting system has the form

$$\frac{\partial f^e}{\partial t}+\frac{eE}{m}\frac{\partial f^e}{\partial v_z}=\frac{\partial}{\partial v_\alpha}D_{\alpha\beta}\frac{\partial f^e}{\partial v_\beta}+St(f_e); \tag{3.5}$$

$$\frac{\partial N_\kappa}{\partial t}=(2\gamma_\kappa-\gamma)N_\kappa, \tag{.6}$$

$$\gamma=\gamma_i+\gamma_{st}. \tag{3.7}$$

where

$$\gamma_i=2\pi^2\kappa\left(\frac{\omega}{\kappa}\right)^4 f_i\left(\frac{\omega}{\kappa}\right)$$

describes the Landau attenuation due to ions, and

$$\gamma_{st} = \frac{1}{2\sqrt{\pi}} \left(\frac{m T_e}{M T_i} \right)^{1/2} \frac{c_s^2}{(\omega_s/\kappa)^2} \nu_{Te} -$$

describes the attenuation of sound due to ion-ion collisions [21],

$$\nu_{Te} = 4\pi e^4 n L / m^2 v^3 -$$

is the frequency of electron-ion collisions. /31

The expressions $D_{\alpha\beta}$ and γ_κ are defined as before by (3.2) and (3.4). As we can easily see, here the only extra terms are those describing collisions in the kinetic equation for electrons (3.1) and in the kinetic equation for ion-sound plasmons (3.3).

In contrast with [4] when collisions are taken into account there is a quasi-stationary solution for the density of ion-sound noise $N(\vec{\kappa})$. We shall write these equations as given in [7]:

$$N(\vec{\kappa}) = \frac{(2\pi)^3}{\omega_\kappa} \frac{\delta(\kappa - \kappa_0)}{\kappa_0^2} W_0(x), \tag{3.8}$$

where $x = \cos\theta = \kappa_z/\kappa$.

$$W_0(x) = \varepsilon_0 \quad \begin{matrix} \text{for} & x \geq x_0 \\ \text{for} & x \leq x_0 \end{matrix} \qquad x_0^2 = \frac{v_0}{u_T} \tag{3.9}$$

$$\varphi(x) = \frac{1}{x(1-\lambda x)^2} \left\{ (x^2 - x_0^2)\left[(x^2 - x_0^2) + 3x^2(1-\lambda x)\right] - \right.$$
$$\left. - \frac{3}{4} \lambda x_0^2 \ln \frac{x + \sqrt{x^2 - x_0^2}}{x_0} \right\}$$
$$\varepsilon_0 = \frac{\lambda}{4\pi^3} \frac{m}{M} (e E n / \kappa_0 s_0^3) \frac{1}{f_e^0(0)}$$

(3.10) (con't. on next page)

$$u_T = 3eE/4\pi v_e^3 f_e^0(0) m \vartheta_T$$
$$\lambda = \frac{s_0}{v_0} = \frac{\omega_s(\kappa_0)}{\kappa_0 v_0}$$

(3.10)
(con't.)

$$v_0 = s_0\left[1 + \gamma(\kappa_0)/\gamma_s^{(0)}(\kappa_0)\right] = \text{const}$$
$$s_0 = \omega_s(\kappa_0)/\kappa_0,$$

Here κ_0 is the root of an equation /32

$$\frac{\partial}{\partial \kappa}\left\{ \gamma_s^0(\kappa)\left[\frac{v_0 \kappa}{\omega(\kappa)} - 1\right] - \gamma(\kappa)\right\} = 0. \qquad (3.11)$$

Furthermore, [7] shows that, just as in the case when collisions were not taken into account, a plane current-voltage characteristic is valid within a certain time interval, i.e., the resistance of a plasma turns out to be directly proportional to the applied field. In addition, within a certain quasi-stationary stage there occurs an increase of the electron and ion temperatures, which results in an increase of the number of escaping electrons, and a cessation of the stationary state which is then no longer described by the system of equations given above.

It should be emphasized that neither in [7] nor in [4] was the deceleration of the ion gas by the ionic-sound noise ever considered, and consequently, the results obtained in [7] are valid in all cases for the time $t < t_0$.

The next step in development of the theory of turbulent plasma in a constant electric field was [8] which is a continuation of [7].

Reference [7], because of its neglect of the anisotropy of the ion distribution function, strictly speaking, contains valid results only for the /33 case when frequency of ion-ion collisions is so high that the principal role is played by the collision absorption of waves, rather than Cherenkov absorption. Reference [8] generalizes the results obtained in [7] to include the case when not only the collision absorption, but also the resonance absorption of waves by ions is important.

The system of equations in [8] as compared with the system (3.5) - (3.7) additionally includes the kinetic equation for resonance ions and the equation for the mean energy of thermal ions.

In the paper equations are derived describing the processes of heating the electron and ion plasma components. In addition, emphasis was laid on the question of heating resonance ions (having velocities v greater than the minimum phase velocity of sound waves) whose detailed analysis cannot be done without taking into account the anisotropy of the ion absorption, and requires a simultaneous solution of the equations for both the electron and the ion distribution functions. The paper shows that under certain conditions (for sufficiently strong electric fields) the process of heating resonance ions may lead to the appearance of escaping ions and electrons of high energy.

It is necessary to point out, first of all, that when solving the problem in [8] and that stated in [7], the nonlinear interaction of waves among themselves was completely neglected, and the solution for the wave spectrum was assumed to have the form (3.8), that of a δ function. In addition, the uniqueness of this type of solution was not shown.

Secondly, the plane current-voltage characteristic obtained in [8], when the noise energy increases linearly is, strictly speaking, valid only when $t < 1/\gamma_{nonlin}$.

Within the framework of equations used in [8] an explanation was made of one important fact. If the electric field satisfies the inequality /34

$$E < E_{Dr}\sqrt{\frac{m_i}{m_e}}\frac{c_s}{c_0}, \tag{3.12}$$

where $c_0 \sim 6 v_{Ti}$, then the number of escaping electrons is exponentially small; however, if the field is sufficiently high and the condition (3.12) is not satisfied, then the distribution function will be far from Maxwellian in the entire velocity range, and an increase of the electron temperature will be accompanied by the appearance of a large number of escaping electrons during a time $t_0 \simeq m_e v_e^3/eEc_0$.

An analogous assertion was also made concerning ions: for $E \gtrsim E_D (m_e/m_i)^{1/6}\frac{c_s}{c_0}$ the escaping ions appear when $t \sim t_i = t_0(m_i/m_e)^{1/3}$. It must be emphasized that, as a number of authors state, in reality escape may not be observed due to mechanisms that were not taken into account in the theory of [8] (for example, braking by Langmuir waves which in principle could be excited by escaping electrons).

The nonlinear effects were first considered by Kingsen in [11] (however, the kinetics of ions was not taken into account therein) who continued work done in [4]. Kingsen assumed the existence of a quasi-stationary solution for the spectrum of the ionic-sound turbulence in the form:

$$N_K = \frac{N_0}{(\theta^2/2 + \varepsilon(\theta))^2}\,\delta(\kappa - \kappa_0 + \Delta\kappa(\theta)) \tag{3.13}$$

Then to find v, ε and $\Delta\kappa(\theta)$, he used the equations

$$\gamma_\Lambda + \gamma_{Hen} = 0 \qquad \frac{\partial}{\partial K}(\gamma_\Lambda + \gamma_{Hen}) = 0,$$

where γ_Λ is the linear increment, and γ_{Hen} is determined by the equation /35

$$\gamma_{\text{нел}} = \int N_{\kappa_1} \frac{d\vec{\kappa}_1}{(2\pi)^{3/2}} \frac{(\vec{\kappa}\vec{\kappa}_1)^2}{\kappa^2 \kappa_1^2} \frac{T_i}{m_i^2 n} \frac{[\vec{\kappa}\vec{\kappa}_1]^2 \omega_-}{v_{Ti}^3 |\kappa_-|^3} \tag{3.14}$$

with $\kappa_1 \gtrsim 1/\lambda_e$ (the ionic portion of the ionic-sound dispersion curve).

It should be pointed out that the nonlinear damping, accounted for in [11], is responsible for the transfer of momentum to ions, and thus for the stationarity of the process (this is analogour to [8], where "collision" damping was responsible for stationarity). One of the important results obtained in [11] is the conclusion that for any initial nonisothermicity a certain "universal" ratio of temperatures is established in the current plasma:

$$\frac{T_e}{T_i} \sim 2L \equiv 2 \ln \left[\left(\frac{T_e}{T_i}\right)^{3/2} \left(\frac{M}{m}\right)^{1/2} \right]. \tag{3.15}$$

Reference [11] estimates the limit of applicability of the theory developed in [7, 8] when the quasi-stationarity is achieved as a result of collisions:

$$\frac{T_e}{T_i} \gtrsim 2(n_0 \lambda_e^3) \ln \left[\left(\frac{T_e}{T_i}\right)^{3/2} \sqrt{\frac{M}{m}} \right], \tag{3.16}$$

i.e., for $n \lambda_e^3 \sim 10^8$, T_e/T_i is several hundred which can of course hardly be the case under stationary conditions in the magnetosphere of interest to us now. /36

There is still another effect which in principle may be responsible for a transfer of momentum to nonresonance ions, and consequently, for stationarity. This is displacement of phase velocities of ion-sound waves due to a weak inhomogeneity of the plasma, discussed in [11]. The essence of the effect is as follows. In the quasi-classical approximation, when $\left\langle \frac{\nabla n}{n} \frac{\partial \omega}{\partial \kappa} \right\rangle \ll \gamma_\lambda^0$, i.e., when the time during which an instability increases is considerably less than the characteristic damping time related to the inhomogeneity, the distribution function of the plasmons must satisfy Liouville's equation (see, for example, [22]):

$$\frac{\partial N}{\partial t} + \frac{\partial \omega}{\partial \vec{k}} \vec{\nabla} N - \vec{\nabla} \omega \frac{\partial N}{\partial \vec{k}} = 2 \gamma N. \tag{3.17}$$

The last term on the left is related to the fact that the plasmon frequency in the quasi-classical approximation is constant, and in that connection the wave number of plasmons propagating in an inhomogeneous plasma changes. In particular, (3.17) implies

$$k^{-2} + \lambda_e^2(x) = \text{const}$$

We thus find that a plasmon propagating along the density gradient, gets "redder", i.e., its wave number decreases, and the principal damping mechanism for a plasmon is a weak damping by electrons. Conversely, plasmons propagating in a direction where the density decreases, suffer an increase in the wave number k, and rapidly enter a region of k such that a strong damping by resonance ions is present there. An analysis performed in [11] has shown that plasmons in an inhomogeneous plasma, emitted at practically any angle, later return in a direction opposite to the density gradient, and during a time $\tau \sim ka/\omega_{pi}$ enter a region of strong damping. (Here $\nabla n/n \cong a^{-1}$). /37

Thus, a weak inhomogeneity in a plasma can be roughly accounted for by an increment $\gamma_i \sim \omega_{pi}/ka$, and this effect, just like nonlinear scattering by ions, can in principle assure quasi-stationarity.

We shall give a criterion for the case when the effect of an inhomogeneity is more important than nonlinear effects:

$$E \ll \frac{n e \lambda_e}{k a} \sqrt{\frac{m T_i}{M T_e}}, \quad a^{-1} \equiv \frac{\nabla n}{n}. \tag{3.19}$$

Now we shall briefly discuss another nonlinear process which may play an important role in the establishment of a quasi-stationary turbulent state in a plasma with a current. The results obtained in [24] provide a basis for assuming that a disintegration of ionic-sound waves into other ionic-sound waves ($S \to S' + S''$) may be such a process. This process is forbidden under the conditions of a weak nonlinearity [22], when in computing the probability of a disintegration one uses the laws of dispersion, obtained in the linear theory. However, when the nonlinearity is not very weak, corrections to the dispersion law, connected with the presence of intense turbulent pulsations of ion sound result in the possibility of disintegrations of a single ion-sound wave into two. /38

The maximum increment $S \to S' + S''$ of a disintegration, according to [24], in a one-dimensional case is

$$\gamma^{SSS} \sim \lambda_e^2 c_s k^3 \tag{3.12}$$

with

$$W^S/nT \sim \lambda_e^4 k^3 \Delta k.$$

A rough estimate of the form of the spectrum of ionic-sound waves in a quasi-stationary state gives $W_k \sim k^2$, in the region $k \ll 1/\lambda_e$. The disintegration process of ionic-sound waves will always prevail over the process of a nonlinear scattering by ions in the quasi-one-dimensional case studied in [24] (the wave vectors are contained in the cone $\Theta \ll 1$). However, in a three-dimensional case the process $S \to S' + S''$ plays a significantly lesser role, since the scattering at large angles occurs only in a relay-like fashion (i.e., by means of a large number of scattering events involving small scattering angles).

One may expect that the disintegration process $S \to S' + S''$ will lower the level of ionic-sound waves, transfering them into a region of absorption by ion-ion collisions [25]. By taking account of processes of this type, the conductivity of a plasma in a strong electric field (especially for $E >$ $E^{**} = \frac{m}{M}\sqrt{4\pi n T_e}$) will be higher than according to [5, 11].

We shall mention still another possible nonlinear process occurring when /39
there is an external magnetic field acting on a plasma. This process may have an important influence on the form of the spectrum of the ionic-sound waves in the region $k \sim 1/\lambda_e$. The process we have in mind involves a decomposition of the ionic-sound waves (S) into ionic-sound waves (S') and electronic-cyclotron longitudinal waves (e) whose dispersion is $\omega = \omega_{He}\cos\theta$, $S \to S' + e$. Under the conditions $\omega_{pi} > \omega > \omega_{He}$, according to [33], this decomposition has the increment

$$\boxed{\gamma^{ss e} \sim \omega_s \frac{W}{nT} \frac{1}{64\pi^2} \frac{k}{\Delta k}\left(\frac{\omega_{pe}}{\omega_{He}}\right)^2.} \tag{3.13}$$

Here, according to the analysis, the decomposition is quite efficient for ionic-sound waves with $k \sim 1/\lambda_e$.

A rough estimation of the spectral form of the ionic-sound waves in the region $k \sim 1/\lambda_e$, taking account of such decompositions [25], results in the expression

$$W_k \sim 1/k. \tag{3.14}$$

The estimates given here lead us to assert that all these effects must be taken into account when solving a specific problem involving the ionic-sound instability in a current plasma.

§ 4. Turbulent Electroconductivity in an Isothermic Plasma /40

Thus far we have considered turbulent electroconductivity in a nonisothermic plasma, where the ionic-sound instability is possible. Now we shall consider the opposite case $T_e \simeq T_i$ which in practice occurs usually at the very beginning of the process involving the passage of a current due to an external field E_0. In this case the electrons will be freely accelerated until their velocity becomes greater than their thermal velocity. This marks the beginning of development of a hydrodynamic instability of electron-ion oscillations which decelerates the drift of electrons, the latter process being discussed in [3]. Before we make a detailed discussion of the results of papers devoted to this case, we shall note that in these papers plasma oscillations are analyzed within the framework of the so-called adiabatic approximation, i.e., when the acceleration of electrons is neglected. This is permissible only for not very large fields.

In the case

$$\kappa E_0 > 4\pi n_0 e \sqrt{\frac{m_e}{m_i}} \tag{4.1}$$

a harmonic with a wave number κ passes the instability region without a substantial growth ("whistling through"). Thus, if $E > E_{max} = 2 L n_0 e \sqrt{m_e/m_i}$, where L is the dimension of the plasma, instabilities will fail to develop. Reference [26] and also [27] show how oscillations develop for the intermediate case involving a passage from the adiabatic to the nonadiabatic theory of linear oscillations. /41

Now we shall proceed to discuss in detail the results pertaining to turbulent electroconductivity in an isothermic plasma $T_e = T_i$. It will be noted that the results discussed below may also qualitatively apply to the case of a large number of escaping electrons in a nonisothermic plasma.

As we know, as soon as the velocity of electrons relative to ions exceeds the thermal velocity, small fluctuations will grow with time in the plasma. The fluctuations which increase as the time goes on derive their energy from the electrons, thus decelerating them in the process.

Buneman [3] was the first to describe the mechanism responsible for a growth of appreciable concentrations of charged particles out of small fluctuations which is followed by a deceleration of the initial electron drift. His model is essentially as follows. The field which gave rise to the original electron drift is assumed to be turned off, so that there remains only the field of fluctuations $\vec{E}$. The paper discusses only nonrelativistic velocities, the plasma is assumed to be infinite in extent, and the ion-electron collisions are neglected. If the thermal distribution of velocities is neglected, then, as we know, we have the following dispersion equation for perturbations in a system involving two colliding streams of electrons and ions

$$\frac{\omega_{pi}^2}{\omega^2} + \frac{\omega_{pe}^2}{(\omega - \vec{k}\vec{u})^2} = 1. \qquad (4.2)$$

Its solution is $\omega = \omega_r \pm i\alpha$, where ω_r is the frequency, and α is the growth increment, corresponding to a given $\vec{k}\vec{u}$, $\vec{k}$ being the wave vector, and $\vec{u}$ the electron drift velocity. The growth increment has a sharp maximum $\alpha_m = \omega_{pe}(m/M)^{1/3}(\sqrt{3}/2\sqrt[3]{2})$ (peak) at $\omega_{pe} = \vec{k}\vec{u}$ of width $\Delta(\vec{k}\vec{u}) = 3\omega_{pe}\left(\frac{m}{M}\right)^{1/3}$, which corresponds to 0,25 ω_{pe} for a hydrogen plasma. /42

This means that, if thermal velocities are neglected, the fluctuational perturbations in the plasma will increase primarily near $k_m = \omega_{pe}/u$.

The question: what is the value of the drift at which the mechanism of the growth of plasma oscillations begins in the stream for $2\kappa T_e$ comparable with the directional energy of electrons, is solved by Buneman by using a generalized integral dispersion equation which takes into account the Maxwellian velocity distrubution of electrons and ions.

$$g\left(\frac{W}{(2\kappa T/M)^{1/2}}\right) + g\left(\frac{W-u}{(2\kappa T/m)^{1/2}}\right) = \lambda_e^2 k^2, \qquad (4.3)$$

where

$$g(z) = -\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{q \exp(-q^2)}{q - z}\, dq,$$

$$W = \frac{\omega}{k} = \frac{\omega_2 - i\gamma}{k}, \quad \lambda_e = \frac{v_e}{\omega_{pe}}, \quad v_e = \sqrt{\frac{T_e \varkappa}{m_e}}$$

Plotting $\zeta\left(\frac{W}{(2\varkappa T/M)^{1/2}}\right)$ and $\lambda_e^2 k^2 - g\left(\frac{W-u}{(2\varkappa T/m)^{1/2}}\right)$, expressed in terms of elementary functions, and the function $\int_0^z e^{z^2} dz$ in a complex domain, Buneman extracted from the diagram thus obtained the following numerical results: /43

1) At any drift velocity only those fluctuations may increase whose wavelength is greater than λ_e (λ_e is the Debye length).

2) An increase in the fluctuations will occur if the drift velocity is greater than $0.9261 v_e\left[1+\sqrt{\frac{m}{M}}\right]$, $v_e = \sqrt{T_e/m_e}$ i.e., the kinetic energy of drifting electrons must reach 0.90 $\varkappa T$, in order for the growth to begin (and not greater than $\varkappa T$, as is usually assumed).

For wavelengths less than $8.32\lambda_e$ and drift energies less than $0.9\varkappa T$, the Landau damping dominates over the mechanism involving the increase in the amplitude.

Then the question was discussed: from what level of the fluctuation intensity does the process of their growth begin?

The heat in the plasma is distributed both among $6N$ degrees of freedom of electrons and ions, and among "free" plasma oscillations. The number of degrees of freedom in undamped plasma oscillations is estimated as

follows. Let us assume that before the acceleration due to the electric field takes place, oscillations at all wavelengths occur in the plasma. In view of the presence of a sufficient number of electrons with velocities $v \leq 3v_{Te}$, in the Maxwellian velocity distribution, the wavelengths of length $\lambda \leq 2\pi \cdot 3 v_{Te}/\omega_{pe}$ will be subject to Landau damping whose physical cause is the resonance between the phase velocity of the wave and the velocity of an appropriate group of electrons. Wavelengths of greater length will not in practice be subject to Landau damping, since the number of particles with velocities $v > 3v_{Te}$ in the Maxwellian tail is negligibly small, let alone the fact that a continuous model of a velocity distribution is not applicable in this region. /44
Hence, plasma oscillations with the phase velocities $v_{\varphi} > 3v_{Te}$ may be considered to be free modes, which permits the wave numbers

$$k < \frac{1}{3\lambda_e}.$$

On the other hand, if one considers a plasma occupying a cube of side cm, the wave numbers should be multiples of $2\pi/L$ cm^{-1}, i.e., in the k-space of wave numbers, the points that correspond to discrete k must lie at distances $(2\pi/L)$ from each other which gives

$$\frac{4}{3}\pi\left(\frac{1}{3\lambda_e}\right)^3\frac{L^3}{(2\pi)^3} = \frac{4\pi}{3}L^3(6\pi\lambda_e)^{-3}$$
$$(L = 1 \text{ cm})$$

as the number of free oscillators each of which carries an energy $\varkappa T$ (kinetic and field energy).

If one assumes that in practice the fluctuations with $k \approx \omega_{pe}/u$ will increase in the interval

$$\delta k = 3\omega_{pe}\left(\frac{m}{M}\right)^{1/3}u^{-1} \simeq \frac{1}{4}\frac{\omega_{pe}}{u}$$

(for H), then from the entire κ-sphere one must separate out only a layer of thickness $\delta\kappa$ and a radius approximately equal to $1/3\lambda_e$, since the distance from the center of the κ-sphere to a spherical layer taken approximately as a disk is $\frac{\omega_{pe}}{u} \ll 1/3\lambda_e$ (the latter being the radius of the κ-sphere), since $u \gg v_{Te}$.

In the disk under consideration, in the κ-space there are $(\pi\delta\kappa/9\lambda_e^2)$ $(1/2\pi)^3$ free modes, each with an energy $\varkappa T$. /45

Hence, the energy of fluctuations capable of rapid increase is $\frac{\delta\kappa}{18}\varkappa T$ $(2\pi\lambda_e)^{-2}$.

The ratio of the energies of fluctuations, capable of amplification, to the energy of directed motion (per unit volume)

$$\mu = \frac{\delta\kappa}{18}\,\frac{\varkappa T}{(2\pi\lambda_e)^2} \Big/ 0{,}5\,N m u^2$$

gave for hydrogen $\frac{1}{6N}(\omega_{pe}/2\pi u)^3$, i.e., about $0.6 \cdot 10^{-8}$ for a "typical" example $N \sim 10^{15}$ cm^{-3}, $mu^2/2 \sim 250$ eV.

Buneman assumes that the time required for full acceleration is equal to the time that it takes for the energy of turbulent pulsations to equal the energy of directed motion. Since the energy of turbulence increases as , where α_m is the maximum value of the growth increment $0.056\,\omega_{pe}$ for hydrogen, then the deceleration time is equal to 27.4 which is the plasma period (deceleration time is computed from the equation $\mu \exp(2\alpha_m t) = 1$). A more accurate calculation, taking into account a correction to α_m depending on κ, followed by integration over $d\kappa$ gave 28.4 plasma periods for a "typical" example considered.

It must be noted that a calculation of the growth of turbulent pulsations is made under an assumption $u = const$, but, on the other hand, on the basis of this calculation a conclusion is obtained about the decrease of n as the pulsations increase. Thus, the calculation is approximate. /46

Considering that the picture of initial fluctuations in the κ-space which then begin to increase is rather difficult to visualize, Buneman separately considers the process of a development of a local perturbation (one-dimensional), having the form $\delta(x)=(2\pi)^{-1}\int e^{-i\kappa_1 x}d\kappa_1$. During a time t the perturbation will have the form:

$$f(x,t)=\frac{1}{2\pi}\int e^{i\left[\omega(\kappa)t-\kappa x\right]}\,d\kappa,$$

where $\omega^{-}(\kappa)$ is the "increasing" solution of the dispersion formula.

Using the method of steepest descent, one finds the first term in the exponent which is independent of κ. This term essentially determines $f(x,t)$.

$$-i\omega_{pe}\frac{x}{u}+\frac{3}{2}ie^{-i\frac{\pi}{3}}\left[\omega_{pi}^{2}\,\omega_{pe}\,(ut-x)^{2}\,x\right]^{1/3}\frac{1}{u} \qquad (4.4)$$

If we consider the term quadratic in κ in the exponent, we obtain a factor which depends weakly on x, t.

The last expression makes it clear that the logarithm of the perturbation's amplitude increases as $(ut-x)^{2/3}\,x^{1/3}$, i.e., as time goes on, the perturbation spreads out between its initial position and the position $x=ut$ which is where the electrons move. As the perturbation spreads, it increases everywhere (although not equally). The peak of the perturbation moves with a /47
velocity $(1/3)u$.

The origin of the coordinates in this process is never free of the initial perturbation which means that it is impossible to use this mechanism for a controlled amplification (of course, if ions do not move in the same direction as electrons).

A study of the imaginary part of Equation (4.4) shows that it approaches $\omega_{pe} x/u$, i.e., during the course of the perturbation it is periodic with a period $2\pi u/\omega_{pe}$.

During the last few periods when the perturbations increase (and, properly, a substantial portion of the directed energy is converted into the energy of fluctuations) the nonlinear effects, involving the deceleration of the stream and an interaction among waves, become very important.

The estimation of relationships between the perturbations of velocities and densities for ions and electrons at a frequency corresponding to the maximum increasing solution, lead Buneman to conclude that nonlinearities in the motion of ions may be neglected.

The main effect of the nonlinear process is the tendency toward isotropy. (In the case of a weak nonlinearity, see [30], [12]). A fairly sharp spectrum with $k = \omega_{pe}/u$, produced during a linear increase, will spread out, and ultimately isotropic conditions will be established, and the drift energy will be distributed equally among all possible plasma oscillators.

In a one-dimensional case, a numerical calculation was made on a computer that confirmed the above assertion.

Qualitatively, "turbulent" mixing occurs in the following way.

The electron layers oscillate and produce a wall of space charge which /48
reflects the electrons. Thus, "collective interactions" take place.

It must be noted that the electrons recoil from their own fluctuation fields (the ions are uniformly distributed).

The first electron wall is scattered within a short time, then it intensifies again and causes further reflections. Some electrons that had already been reflected once are reflected again from the second wall, and thus are "trapped".

Other electrons have enough energy to pass all walls in both directions. The mixing process results in the appearance of many currents.

A case which is more important in practice involves a situation in which the drift is produced by a uniform external electric field. If, according to the mechanism described, the energy of the directed motion is converted into random energy, but the constant field "stretches" the drift, then the time required for a new "disruption" of the drift is on the order of the relaxation time, and is equal to $2\pi/\omega_{pe}$ which is the plasma period. Consequently, the "frequency" of the collective interactions is $\nu^{coll} \simeq \omega_{pe}/\beta$ (where $\beta \sim 10$)), and the constant velocity can be found from the relation $m u \omega_{pe}/\beta = eE$.

Hence, the conductivity is equal to

$$\sigma = \frac{\beta e^2 N}{m\omega_{pe}} = \frac{\beta}{4\pi}\omega_{pe}.$$

Let us consider in more detail the case when the field of the measured quantity /49 slowly increases the translational velocity $u(t)$ at the same time as the increasing turbulent pulsations ultimately result in a deceleration and an establishment of $u = const$.

The criterion for the applicability of the "adiabatic" theory we are discussing is:

$$\frac{eE}{m\alpha_m} \ll \frac{\omega_{pe}}{\kappa} = u.$$

The curve $\alpha(\kappa_\parallel)$, having a maximum α_m at $\kappa u = \omega_{pe}$, is replaced in rough estimates by a "rectangular" function, i.e., the wave numbers κ near the value ω_{pe}/u in an interval of width $0.25\,\omega_{pe}/u$ have a growth increment α_m and zero outside of the interval.

The amplification of individual wave numbers continues only as long as $u(t)$ is inside the interval $\Delta u = 0.25\,\omega_{pe}/\kappa$ about ω_{pe}/κ. At other times one can say that the wave number is beyond resonance.

The continuation of the amplification of a fluctuation with a wave number κ is limited to the time $\Delta t = \Delta u/(eE/m)$, and the fluctuation energy is amplified by e^{τ}, where

$$\tau(\kappa) = 2\alpha_m\,(0.25\,\omega_{pe}/\kappa)\frac{m}{eE}. \tag{4.5}$$

At a time t the waves in the neighborhood of $\kappa = \omega_{pe}/u(t) = m\omega_{pe}/eEt$ are unstable. First waves with large κ become unstable, and by the time $t = t_1$ all $\kappa > m\omega_{pe}/eEt_1$ will pass through the time interval in which amplification occurs. Thus, the minimum $\tau = 0.5\alpha_m t_{max}$ will be achieved by later and the smallest κ.

The upper limit of κ is determined by the Debye length $\kappa_{max} = 2\pi/\lambda_e$. /50
However, often an even more serious limitation yields a condition of adiabaticity, $\kappa \ll \omega_{pe} m\alpha_m/eE$. Furthermore, integrating over all wave numbers, which result in the exponent ranging from 1 to τ, we shall find the total increase in energy.

The initial fluctuation energy per unit volume amounts to $(\delta\kappa/18)\,\varkappa T\,(2\pi\lambda_e)^{-2}$ which is $\omega_0\delta\kappa = \delta\kappa\,(e^2/8\pi)$ per single electron. Using (4.5), we can express $\delta\kappa$ in terms of $\delta\tau$,

$$\delta\kappa = -\frac{1}{2} m\omega_{pe}\alpha_m \delta\tau/\tau^2 e E$$

thus obtaining the total turbulent energy per electron:

$$W_{\text{турб}} = \int w^s \, \delta\kappa \, e^{\tau(\kappa)} = \frac{e m \omega_{pe}\alpha_m}{36\pi E}\int_1^{\tau_1} \exp(\tau)\frac{d\tau}{\tau^2}. \tag{4.6}$$

For large $\tau_1 = \frac{t_1\alpha_m}{2}$ the integral is not sensitive to the lower limit, and is equal to $\exp\tau_1/\tau_1^2$. For hydrogen (denoting by $E_0 = e/\ell^2 = eN^{2/3} = 1.44\,\ell^{-2}\,10^{-7}\,\frac{V}{cm}$ the Coulomb field of the nearest neighbor in a cubic latice, and by W the energy of interaction with the nearest neighbor), we have

$$W_{\text{турб}} = \frac{1}{160} W \frac{E_0}{E} e^{\tau_1}\frac{1}{\tau_1^2}. \tag{4.7}$$

During that time the energy of directed motion will increase to /51

$$W_{\text{др}} = \frac{1}{2}(eEt_1)^2/m = 2e^2E^2\tau_1^2/m\alpha_m^2 = 50\,W\left(\frac{E}{E_0}\right)^2\tau_1$$

(for hydrogen). Hence, the condition for the drift to be halted by fluctuations is

$$\boxed{\left|20\,\frac{E}{E_0}\right|^3\tau_1^4 = \exp\tau_1,} \tag{4.8}$$

whence, $\tau_1 \sim 3\ln|20E/E_0|$.

Numerical estimates show that the time required for establishment of a constant turbulent current is on the order of $100\ 2\pi/\omega_{pe}$ for a wide range of applied fields.

The estimate of the time during which the acceleration may occur without disturbances for weak fields must be modified. Otherwise, the approximation (4.6) is invalid for small t. A major portion of the time of acceleration may fall in the stable interval $\frac{mu^2}{2} < 0.9\varkappa T$, and then weak fields may not be able to accelerate electrons to 0.9 kT without collisions. For the electron-ion plasma, the requirement that we neglect close interactions is reduced to $E > E_{sp} \cong a/(2\lambda e)^2$, for — if the electrons are accelerated without collisions to $\overline{\varkappa T}$, — then they already are able to escape. /52

The mechanism we are discussing brings the "escaping" electrons to a halt by way of collective interactions. The conductivity is estimated here as

$$\sigma = \frac{evn}{E} = \frac{eE}{m}\frac{\lambda}{v_{st}}\frac{en}{E} = 50\omega_{pe}.$$

Thus, the essential result of Buneman's paper was to give a qualitative picture of "electrodynamic" heating. A strong field produces drift within ~ 100 plasma periods. This is followed by turbulization, and the energy of directed motion is converted into heat. Then the directed velocity is re-established, but as soon as the energy of directed motion reaches a value corresponding to a new "temperature", the translational motion breaks down, etc.

Thus, the temperature (a generalization of the usual equilibrium concept) increases, and a drift elevation of random energy never lasts very long.

A more detailed discussion of the process of deceleration of a beam of "escaping" electrons in a strong electric field $E \gg E_{sp}$ effected by turbulent pulsations was given by Shapiro [28] who used the hydrodynamic approximation.

The initial system of equations for the problem consists of the Boltzmann-Vlasov equations for electrons and ions and of Poisson's equation:

$$\frac{\partial f^{e,i}}{\partial t} + \vec{v}\frac{\partial f^{e,i}}{\partial \vec{r}} \mp \frac{e}{m^{e,i}}(\vec{E}_0+\vec{E}_1)\frac{\partial f^{e,i}}{\partial \vec{v}} = 0, \tag{4.9}$$

$$\mathrm{div}\,\vec{E}_1 = 4\pi e\left\{\int f^{i} d\vec{v} - \int f^{e} d\vec{v}\right\}. \tag{4.10}$$

We introduce the assumptions that the distribution functions may be written as /53

$$f^{e,i}(t,\vec{r},\vec{v}) = f_0^{e,i}(t,\vec{v}) + f_1^{e,i}(t,\vec{r},\vec{v}), \tag{4.11}$$

where

$$\langle f_1^{e,i}(t,\vec{r},\vec{v})\rangle = \frac{1}{V}\int f_1^{e,i} d\vec{r} = 0, \tag{4.12}$$

and V is an arbitrary microscopic volume.

E_1 can be written as

$$E_1(t,\vec{r}) = \sum\left(\varepsilon_k(t)\cos\vec{k}\vec{r} + \varepsilon'_k(t)\sin\vec{k}\vec{r}\right). \tag{4.13}$$

Using (4.11) and (4.12) it was found in [28] that

$$\frac{\partial f_0^{e,i}}{\partial t} \mp \frac{e}{m^{e,i}}\vec{E}_0\frac{\partial f_0^{e,i}}{\partial \vec{v}} \mp \frac{e}{m^{e,i}}\left\langle \vec{E}_1 \frac{\partial f_1^{e,i}}{\partial \vec{v}}\right\rangle = 0 \tag{4.14}$$

and, furthermore, multiplying this by $m^{e,i}\vec{v}$ and integrating over the velocities, the equation for the translational momentum of electrons and ions in the plasma is

$$\frac{d\vec{P}_0^{e,i}}{dt} = \mp e n_0 \vec{E}_0 \mp e\langle n_1^{e,i}\vec{E}_1\rangle$$
$$n_1^{e,i} = \int f_1^{e,i}\, d\vec{v}. \qquad (4.15)$$

In discussing plasma oscillations, Shapiro used the usual dispersion equation for the case of a constant drift which is applicable only when the adiabaticity condition is satisfied /54

$$\dot{u}_0^e \ll \omega u_0^e. \qquad (4.16)$$

The entire discussion assumed the condition of linearity of oscillations $eE_\kappa/m\omega u_0 \ll 1$ (E_κ is the amplitude of oscillations with a wave number κ).

Furthermore, Shapiro made an approximate calculation of the decelerating force

$$\vec{F}^e = -e\langle n_1^e\vec{E}_1\rangle = \frac{-e}{V}\int \sum_\kappa \vec{E}_1^\kappa \sum_{\kappa'} n_1^{e\kappa'}\, d\vec{r} \qquad (4.18)$$

using the hydrodynamic equations of motion, continuity, and Poisson's equation, as well as the dispersion equation (4.1) which gives a complex frequency of oscillations, $\omega - i\delta$ (the increasing solutions are used). The initial amplitude of oscillations was determined from the condition

$$VE_\kappa^2/8\pi = \varkappa T_{in}. \qquad (4.19)$$

As a result of laborious calculations, an equation was obtained for the translational momentum in the form

$$\frac{dp_{ox}^{e}}{dt} = -en_0 E_0 \left\{ 1 + \frac{m}{M} \frac{64\pi\sqrt{2\pi}}{|\alpha''(y^e)|^{1/2}} \left(\frac{E_g}{E_0}\right)^3 \times \right.$$

$$\left. \times \frac{1}{y^e} \frac{X_2^e X_i^e}{(X_2^{e2}+X_i^{e2})^2} \left[\frac{y^{e2}}{(y^e-X_2^e)^2} + \frac{X_2^{e2}-X_i^{e2}}{X_2^e y^e}\right] \frac{e^{\alpha(y^e)\tau}}{\tau^{5/2}} \right\} \tag{4.20}$$

$$E_g = eN_0^{2/3},$$

$$\tau = \omega_{pe} t, \quad \alpha(y) = \frac{2}{y} \int_0^y X_i(y')dy',$$

where

$$X_2^e = \frac{\omega(y^e)}{\omega_{pe}}, \quad X_i^e = \frac{\delta(y^e)}{\omega_{pe}}, \quad y = -\frac{K_x u_0^e}{\omega_{pe}},$$

y^e is determined from the equation $\alpha'(y) = 0$, $p = \omega - i\delta$ is the solution /55
of the dispersion equation (4.1).

An approximate integration of Equation (4.20) for the hydrogen plasma gives

$$p_{ox} = -\frac{en_0 E_0}{\omega_{pe}} \left\{ \tau - \left(\frac{m}{M}\right)^{1/3} 0.2\pi^{3/2} \left(\frac{E_g}{E_0}\right)^3 \frac{e^{\alpha\tau}}{\alpha\tau^{5/2}} \left[1 + \frac{5}{2\alpha\tau} + \cdots\right] \right\} \tag{4.21}$$

(α_e =0,040) for $\frac{1}{\alpha_e} \ll \tau \leq \tau_e$, where τ_e/ω_{pe} is the time when the maximum momentum is achieved.

From the above expression it is clear that the electron gas is initially accelerated linearly with time, and then it begins to be decelerated by interactions with turbulent pulsations.

The maximum momentum of the electron flux is attained at a time which can be determined from the equation

$$1 = 0.2\,\pi^{3/2}\left(\frac{m}{M}\right)^{1/3}\left(\frac{E_g}{E_0}\right)^3 \frac{e^{\alpha\tau_e}}{\alpha\tau^{5/2}} . \qquad (4.22)$$

The maximum value of the translational electron momentum is determined by the formula /56

$$p_{0x_{max}} = -\frac{e n_0 E_0}{\omega_{pe}}\left\{\tau_e - \frac{1}{\alpha}\left(1+\frac{5}{2\alpha\tau_e}\right)\right\} \approx e n_0 E t_e . \qquad (4.23)$$

This corresponds to a maximum conductivity of the plasma

$$\sigma_{max} \simeq 50\,\omega_{pe} . \qquad (4.24)$$

For $t > t_e$, Equation (4.20) becomes inapplicable because of an increase in the electron temperature and a decrease of the growth increment, and an appearance of nonlinear effects.

Estimates made by Shapiro showed that at $t \sim t_e$ the energy of plasma oscillations is much smaller than the translational energy; and consequently, at that time the amplitudes of the oscillations are small as compared with those at which there occurs a pronounced "nonlinear mixing".

Furthermore, then Shapiro [29] discusses the same problem about the deceleration of electrons by turbulent pulsations, but directly on the basis of the kinetic equation.

Making in Equation (4.13) a substitution

$$w = v - u_0^{\mp} \quad \left(u_0^{\mp} = \mp\frac{e}{m_{\mp}}\int_0^t E_0\,d\tau\right)$$

the equation becomes

$$\frac{\partial f^{\mp}}{\partial t} \mp \frac{e}{m_{\mp}} \left\langle \vec{E}_1 \frac{\partial f^{\mp}}{\partial \vec{w}} \right\rangle = 0 \tag{4.25}$$

Substituting in this equation the expansions: /57

$$E_1 = \sum_{k} E_k \exp\left\{i\left(\vec{k}\vec{r} - \int_0^t \omega_k \, d\tau\right)\right\},$$
$$f_1^{\mp} = \sum_{k} f_k^{\mp} \exp\left\{i\left(\vec{k}\vec{r} - \int_0^t \omega_k \, d\tau\right)\right\}$$

and using the usual formulas of the linearized theory for oscillations in the plasma

$$f_k^{\mp} = \pm \frac{e}{m_{\mp}} \vec{E}_k \frac{\partial f_0^{\mp}(t, \vec{w})}{\partial \vec{w}} \frac{1}{i\left[\vec{k}(\vec{w} + \vec{u}_0^{\mp}) - \tilde{\omega}_k\right]},$$

where $\tilde{\omega}_k = \omega_r - i\delta_k$ is the solution of the dispersion equation, Shapiro [29] obtained an equation for $f_0^{\mp}$:

$$\frac{\partial f_0^{\mp}}{\partial t} = \frac{\partial}{\partial \vec{w}}\left(\vec{I}^{\mp} f_0^{\mp}\right) + \frac{1}{2} \frac{\partial^2}{\partial w_i \partial w_j}\left(\beta_{ik}^{\mp} f_0^{\mp}\right) \tag{4.26}$$

(of the Fokker-Planck type).

$\vec{I}^{\mp}$ and β_{ik} are rather complicated integrals, depending on the parameters w, u_0, t.

The equation for Fourier components can be solved using the method of successive approximations, where w/u_0 is the expansion parameter. Then a fairly complicated distribution function is found which has different longitudinal and lateral time-dependent temperatures:

$$\Theta_\perp(t) = \frac{\Theta_0}{2}\left(1-\gamma+\sqrt{(1-\gamma)^2+4\delta}\right);\ \Theta_\parallel(t)=\Theta_0(1+2\gamma);$$
$$\gamma = 18\pi^{3/2}(E_g/E_0)^3\frac{e^{\alpha_e\tau}}{\tau^{3/2}};\ \delta=\frac{1.4}{\pi^{5/2}}\frac{E_D}{E_0}\frac{e^{\alpha_e\tau}}{\tau^{3/2}} \tag{4.27}$$
$$(\alpha_e\tau \gg 1)\quad E_D = e/\lambda_e^2.$$

The growth of thermal energy in the plasma is related to Landau damping. /58

The most interesting result of Shapiro's paper is the fact that temperature anisotropy is achieved with increasing t.

The equation for the translational momentum $p_z = m_-\int w_z f_0(t,w)\,d\vec{w}$ is therefore modified:

$$\frac{dp_z}{dt} = -eE_0 n_0\left\{1-\frac{1.4}{\sqrt{\pi}}\frac{\Theta_0}{\Theta_\perp}\left(\frac{E_g}{E_0}\right)^3\frac{e^{\alpha_e\tau}}{\tau^{5/2}}\right\}, \tag{4.28}$$

and the maximum momentum of the electron stream is attained at $\tau = \tau_1$, which can be determined from the equation

$$1 = \frac{1.4}{\sqrt{\pi}}\frac{\Theta_0}{\Theta_\perp(\tau_1)}\left(\frac{E_g}{E_0}\right)^3\frac{e^{\alpha_e\tau_1}}{\tau_1^{5/2}}. \tag{4.29}$$

As before, after a time τ_e the entire theory is strictly speaking inapplicable. Practically speaking for $\tau \gg \tau_e$, the plasma becomes nonisothermic, and the process is described by the theory presented in the preceding section.

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